

Decoherence in an infinite range Heisenberg model

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Abstract. We study decoherence in an infinite range Heisenberg model (IRHM) in the two situations where the system is coupled to a bath of either local optical phonons or global optical phonons. Using a non-perturbative treatment, we derive an effective Hamiltonian that is valid in the regime of strong spin-phonon coupling under non-adiabatic conditions. It is shown that the effective Hamiltonian commutes with the IRHM and thus has the same eigenstates as the IRHM. By analyzing the dynamics of the system using a quantum master equation approach, we show that the quantum states of the IRHM system do not decohere under Markovian dynamics when the spins interact with local phonons. For interactions with global phonons, the off-diagonal matrix elements of the system's reduced density matrix, obtained for non-Markovian dynamics, do not indicate decoherence only when states with the same S_T^z (i.e., eigenvalue for the z-component of the total spin) are considered.

1. Introduction

Quantum information processing heavily relies on a precious and fragile resource, namely, quantum entanglement [1]. The fragility of entanglement is due to the coupling between a quantum system and its environment; such a coupling leads to decoherence, the process by which information is degraded. Decoherence is the fundamental mechanism by which fragile superpositions are destroyed thereby producing a quantum to classical transition [2, 3]. In fact, decoherence is one of the main obstacles for the preparation, observation, and implementation of multi-qubit entangled states. The intensive work on quantum information and computing in recent years has tremendously increased the interest in exploring and controlling decoherence effects [4, 5, 6, 7, 8, 9, 10, 11].

Since coupling of a quantum system to the environment and the concomitant entanglement fragility are ubiquitous [1, 2], it is imperative that progress be made in minimizing decoherence. Decoherence free states prevent the loss of information due to destructive environmental interactions and thus circumvent the need for stabilization methods for quantum computation and quantum information. In the past decoherence-free-subspace (DFS) [12, 13] has been shown to exist in the Hilbert space of a model where all qubits of the quantum system are coupled to a common environment with equal strength. A DFS is a subspace which is invariant under the action of the system Hamiltonian; furthermore, the subspace is spanned by degenerate eigenvectors of the system operators coupling to the environment [14, 15]. Alternately, decoherence can also be suppressed through quantum control strategies [16, 17, 18].

Although the theory of decoherence has undergone major advances [2, 3], yet, there exist many definitions of decoherence [19]. For the analysis in this paper, we choose the most commonly used definition of decoherence: Loss of off-diagonal elements in the system's reduced density matrix. In general, a many-qubit (i.e., many-spin) system can have distance dependent interaction. The two limiting cases for interaction are spin interactions that are independent of distance and spin chains with nearest-neighbor interactions only. In this work we consider the extreme case of distance independent interaction among the spins, i.e., the IRHM. The objective of this paper is to study the decoherence phenomenon, due to coupling of spins of the IRHM to optical phonons, in the two extreme cases of the spins being independently coupled to different baths; and all the spins being collectively coupled to the same environment. We employ the analytically simpler frame of reference of hard-core-bosons (HCBs) rather than that of spins so that the single particle excitation spectrum can be easily obtained and exploited; we show that the effective Hamiltonian even in higher order (i.e., greater than second order) perturbation theory retains the same eigenstates as the IRHM when the spins are coupled to local phonons. Furthermore, decoherence is studied using the quantum master equation approach [20]. Our dynamical analysis shows that the system coupled to local phonons does not decohere when Markov processes are considered; whereas for global phonons, even for non-Markovian dynamics, there is no decoherence

when eigenstates with the same eigenvalue S_T^z (i.e., z-component of the total spin) are considered.

The rest of the paper is organized as follows: In section 2, we introduce the IRHM Hamiltonian and describe its eigenstates and eigenenergies. In section 3, we study decoherence under strong coupling with local optical phonons, and show that the effective Hamiltonian thus obtained retains the same eigenstates as H_{IRHM} . In section 4, we use the master equation approach and show that the system does not decohere under local and global couplings. Next, in section 5, we give our conclusions and make some general remarks regarding the wider context of our results. The paper also contains an appendix where we derive the third order perturbation contribution to our effective Hamiltonian (H_{eff}) and show that the eigenstates of the IRHM Hamiltonian are retained by our H_{eff} .

2. Infinite Range Heisenberg Model

We begin by introducing the IRHM whose decoherence will be studied when the system is coupled to either local or global optical phonons. The IRHM is defined as:

$$\begin{aligned} H_{\text{IRHM}} &= J \sum_{i,j>i} [\vec{S}_i \cdot \vec{S}_j + (\Delta - 1) S_i^z S_j^z] \\ &= \frac{J}{2} \left[\left(\sum_i \vec{S}_i \right)^2 - \left(\sum_i \vec{S}_i^2 \right) \right. \\ &\quad \left. + (\Delta - 1) \left\{ \left(\sum_i S_i^z \right)^2 - \left(\sum_i S_i^{z2} \right) \right\} \right], \end{aligned} \quad (1)$$

where $J > 0$, $\Delta \geq 0$, and we are considering only $S = 1/2$ spins. We note that H_{IRHM} commutes with both S_{Total}^z ($\equiv \sum_i S_i^z$) and $\left(\sum_i \vec{S}_i \right)^2$ ($\equiv S_{\text{Total}}^2$). In equation (1), it is understood that $J = J^*/(N - 1)$ (where J^* is a finite quantity) so that the energy per site remains finite as $N \rightarrow \infty$. The eigenstates of H_{IRHM} are characterized by S_T (i.e., the total spin eigenvalue) and S_T^z (or the eigenvalue of the z-component of the total spin S_{Total}^z); the eigenenergies of these eigenstates are

$$E_{S_T} = \frac{J}{2} \left[S_T(S_T + 1) - \frac{3N}{4} + (\Delta - 1) \left(S_T^2 - \frac{N}{4} \right) \right]. \quad (2)$$

The ground state corresponds to $S_T^z = 0$ and $S_T = 0$ which is rotationally invariant.

3. Effective Hamiltonian for IRHM spins coupled to local optical phonons

The real quantum computer will not be free from noise and thus the entangled states have a tendency to undergo decoherence. To study decoherence due to phonons, we consider interaction with optical phonons such as would be encountered when considering transition metal oxides. We will now derive an effective Hamiltonian,

when the spins of the IRHM are coupled to local optical phonons, and show that the eigenstates of the effective Hamiltonian are the same as those of H_{IRHM} .

The total Hamiltonian H_T is given by

$$H_T = H_{\text{IRHM}} + g\omega \sum_i S_i^z (a_i^\dagger + a_i) + \omega \sum_i a_i^\dagger a_i, \quad (3)$$

where a is the phonon destruction operator [21], ω is the optical phonon frequency, and g is the coupling strength. Now, we make the connection that the spin operators can be expressed in terms of HCB creation and destruction operators b^\dagger and b , i.e., $b^\dagger = S^+$, $b = S^-$, and $b^\dagger b = S^z + 0.5$. We then observe that conservation of S_{Total}^z implies conservation of total number of HCB. The total Hamiltonian is then given by

$$H = J \sum_{i,j>i} [(0.5b_i^\dagger b_j + \text{H.c.}) + \Delta(n_i - 0.5)(n_j - 0.5)] \\ + \omega \sum_j a_j^\dagger a_j + g\omega \sum_j (n_j - \frac{1}{2})(a_j + a_j^\dagger), \quad (4)$$

where $n_j \equiv b_j^\dagger b_j$. Subsequently, we perform the well-known Lang-Firsov (LF) transformation [22, 23] on this Hamiltonian. Under the LF transformation given by $e^S H e^{-S} = H_0 + H_I$ with $S = -g \sum_i (n_i - \frac{1}{2})(a_i - a_i^\dagger)$, the operators b_j and a_j transform like fermions and bosons; this is due to the interesting commutation properties of HCB given below:

$$[b_i, b_j] = [b_i, b_j^\dagger] = 0, \text{ for } i \neq j, \\ \{b_i, b_i^\dagger\} = 1. \quad (5)$$

Next, the unperturbed Hamiltonian H_0 is expressed as [23]

$$H_0 = H_s + H_{\text{env}}, \quad (6)$$

where we identify H_s as the system Hamiltonian

$$H_s = J \sum_{i,j>i} [(0.5e^{-g^2} b_i^\dagger b_j + \text{H.c.}) \\ + \Delta(n_i - 0.5)(n_j - 0.5)], \quad (7)$$

and H_{env} as the Hamiltonian of the environment

$$H_{\text{env}} = \omega \sum_j a_j^\dagger a_j. \quad (8)$$

On the other hand, the interaction H_I which we will treat as perturbation is given by

$$H_I = J \sum_{i,j>i} [0.5e^{-g^2} b_i^\dagger b_j] \{ \mathcal{S}_+^{ij\dagger} \mathcal{S}_-^{ij} - 1 \} + \text{H.c.}, \quad (9)$$

where $\mathcal{S}_\pm^{ij} = \exp[\pm g(a_i - a_j)]$. In the transformed frame, the system Hamiltonian depicts that all the HCBs are coupled to the same phononic mean-field. Thus, the unperturbed Hamiltonian H_0 comprises of the system Hamiltonian H_s representing HCBs with the same reduced hopping term $0.5J e^{-g^2}$ and the environment Hamiltonian H_{env} involving displaced bath oscillators corresponding to local distortions. Here it should be pointed

out that both the interaction of the HCB with the mean-field as well as the local polaronic distortions in the bath oscillators involve controlled degrees of freedom. Now, the system Hamiltonian H_s can be expressed as

$$H_s = H_{\text{IRHM}} + (H_s - H_{\text{IRHM}}) \quad (10)$$

When we change the Hamiltonian from H_{IRHM} to H_s by adiabatically turning on the perturbation $(H_s - H_{\text{IRHM}})$, the resulting state of the system is still obtainable from that of H_{IRHM} by using unitary Hamiltonian dynamics and is thus predictable based on a knowledge of the coupling parameter g [24]. Thus no irreversibility is involved in going from H_{IRHM} to H_s . *On the other hand, perturbation H_I pertains to the interaction of HCBs with local deviations from the phononic mean-field; the interaction term H_I represents numerous or uncontrolled environmental degrees of freedom and thus has the potential for producing decoherence.* Furthermore, it is of interest to note that the interaction term is weak in the transformed frame compared to the interaction in the original frame; thus one can perform perturbation theory with the interaction term.

We represent the eigenstates of the unperturbed Hamiltonian H_0 as $|n, m\rangle \equiv |n\rangle_s \otimes |m\rangle_{ph}$ with the corresponding eigenenergies $E_{n,m} = E_n^s + E_m^{ph}$; $|n\rangle_s$ is the eigenstate of the system with eigenenergy E_n^s while $|m\rangle_{ph}$ is the eigenstate for the environment with eigenenergy E_m^{ph} . Henceforth, for brevity, we will use $\omega_m \equiv E_m^{ph}$. On observing that $\langle 0, 0 | H_I | 0, 0 \rangle = 0$ (i.e., the ground state expectation value of the deviations is zero), we obtain the next relevant second-order perturbation term [23]

$$E^{(2)} = \sum_{n,m} \frac{\langle 0, 0 | H_I | n, m \rangle \langle n, m | H_I | 0, 0 \rangle}{E_{0,0} - E_{n,m}}. \quad (11)$$

For strong coupling ($g > 1$) and non-adiabatic ($J^*/\omega \leq 1$) conditions, on noting that $\omega_m - \omega_0 = \omega_m$ is a positive integral multiple of ω and that $E_n^s - E_0^s \sim J^* e^{-g^2} \ll \omega$ (as shown in the next section), we get the following second-order term $H^{(2)}$ [25] using Schrieffer-Wolff (SW) transformation (as elaborated in Appendix A of references [26] and [27]):

$$\begin{aligned} H^{(2)} &= - \sum_m \frac{{}_{ph}\langle 0 | H_I | m \rangle_{ph} {}_{ph}\langle m | H_I | 0 \rangle_{ph}}{\omega_m} \\ &= \sum_{i,j>i} \left[(0.5 J_{\perp}^{(2)} b_i^{\dagger} b_j + \text{H.c.}) \right. \\ &\quad \left. - 0.5 J_{\parallel}^{(2)} \{ n_i (1 - n_j) + n_j (1 - n_i) \} \right], \end{aligned} \quad (12)$$

where

$$J_{\perp}^{(2)} \equiv -(N-2) f_1(g) \frac{J^2 e^{-2g^2}}{2\omega} \sim -(N-2) \frac{J^2 e^{-g^2}}{2g^2 \omega}, \quad (13)$$

$$J_{\parallel}^{(2)} \equiv [2f_1(g) + f_2(g)] \frac{J^2 e^{-2g^2}}{2\omega} \sim \frac{J^2}{4g^2 \omega}, \quad (14)$$

with $f_1(g) \equiv \sum_{n=1}^{\infty} g^{2n}/(n!n)$ and $f_2(g) \equiv \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} g^{2(n+m)}/[n!m!(n+m)]$. The effective Hamiltonian $H_s + H^{(2)}$ is a low energy Hamiltonian obtained by the canonical

SW transformation [28, 29] decoupling the low-energy and the high-energy subspaces; this decoupling is a consequence of $J^*e^{-g^2} \ll \omega$. We now make the important observation that the effective Hamiltonian $H_s + H^{(2)}$, when expressed in terms of spins, has the following form:

$$\sum_{i,j>i} [J_{\text{tr}}(S_i^x S_j^x + S_i^y S_j^y) + J_{\text{lng}} S_i^z S_j^z], \quad (15)$$

and thus has eigenstates identical to those of the original Hamiltonian H_{IRHM} in equation (1) because $\sum_{i,j>i} (S_i^z S_j^z)$ and H_{IRHM} commute. On carrying out higher order (i.e., beyond second order) perturbation theory (as discussed in Appendix A), and expressing the results in the spin language, we still get an effective Hamiltonian H_{eff} of the following form that has the same eigenstates as the IRHM.

$$H_{\text{eff}} = \sum_{i,j>i} \left[J_{xy} \left(\sum_k S_k^z \right) (S_i^x S_j^x + S_i^y S_j^y) \right] + \sum_i J_z \left(\sum_k S_k^z \right) S_i^z, \quad (16)$$

where J_{xy} and J_z are functions of the $S_{T\text{total}}^z (= \sum_k S_k^z)$ operator. The small parameter of our perturbation theory, for a small N system, is $J/(g\omega)$ [see reference [30] for details]; whereas for a large N, the small parameter is $J^*/(g^2\omega)$ [see reference [31] for an explanation]. It is the infinite range of the Heisenberg model that enables the eigenstates of the system to remain unchanged. Next, we study decoherence in a dynamical context and gain more insight into how the states of our H_{IRHM} can be decoherence free.

4. Dynamical evolution of the system

In this section, we will study decoherence in the system from the dynamical perspective. We will discuss the dynamics of an open quantum system, described by the Hamiltonian IRHM, using master equation approach. Our quantum system is open because it is coupled to another quantum system, i.e., a bath or environment [34]. In our case, IRHM is coupled to a bath of either local optical phonons [see equation (3)] or global optical phonons [see equation (38)]. As a consequence of the system-environment coupling, the state of the system may change. This interaction may lead to certain system-environment correlations such that the resulting state of the system may no longer be represented in terms of unitary Hamiltonian dynamics. The dynamics of the system, described by the reduced density matrix $\rho_s(t)$ at time t , is obtained from the density matrix $\rho_T(t)$ of the total system by taking the partial trace over the degrees of freedom of the environment:

$$\rho_s(t) = \text{Tr}_R [\rho_T(t)] = \text{Tr}_R [U(t)\rho_T(0)U^\dagger(t)], \quad (17)$$

where $U(t)$ represents the time-evolution operator of the total system. Now it is evident from the above equation that we need first to determine the dynamics of the total system which is a difficult task in most of the cases. By contrast, master equation approach

conveniently and directly yields the time evolution of the reduced density matrix of the system interacting with an environment. This approach relieves us from the need of having to first determine the dynamics of the total system-environment combination and then to trace out the degrees of freedom of the environment.

4.1. Decoherence due to Local Optical Phonons:

We begin this sub-section by considering the following Hamiltonian:

$$H = H_0 + H_I, \quad (18)$$

where H_0 is the system-environment Hamiltonian given by equation (6) and H_I represents the interaction Hamiltonian given by equation (9). It is convenient and simple to derive the quantum master equation in the interaction picture. Thus our starting point is the interaction picture von Neumann equation for the total density operator $\tilde{\rho}_T(t)$

$$\frac{d\tilde{\rho}_T(t)}{dt} = -i[\tilde{H}_I(t), \tilde{\rho}_T(t)], \quad (19)$$

where $\tilde{H}_I(t) = e^{iH_0t}H_Ie^{-iH_0t}$ and $\tilde{\rho}_T(t) = e^{iH_0t}\rho_T(t)e^{-iH_0t}$ are the interaction Hamiltonian and the total system density matrix operators (respectively) expressed in the interaction picture. Re-expressing the above equation in integral form yields

$$\tilde{\rho}_T(t) = \tilde{\rho}_T(0) - i \int_0^t d\tau [\tilde{H}_I(\tau), \tilde{\rho}_T(\tau)]. \quad (20)$$

Nowadays there is considerable interest in systems with initial correlation with the environment [32, 33]; however, for simplicity, let us suppose that the initial state of the total system is a factorized state given as $\rho_T(0) = \rho_s(0) \otimes R_0$ with $R_0 = \sum_n |n\rangle_{ph} \langle n| e^{-\beta\omega_n} / Z$ being the initial thermal density matrix operator of the environment and $\beta = \frac{1}{k_B T}$; furthermore, $Z = \sum_n e^{-\beta\omega_n}$ defines the partition function of the environment. With this assumption, we substitute equation (20) inside the commutator of equation (19) and then take the trace over the environmental degrees of freedom to obtain the following equation:

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & -i \text{Tr}_R[\tilde{H}_I(t), \tilde{\rho}_s(0) \otimes R_0] \\ & - \int_0^t d\tau \text{Tr}_R[\tilde{H}_I(t), [\tilde{H}_I(\tau), \tilde{\rho}_T(\tau)]]. \end{aligned} \quad (21)$$

The above equation still contains the total density matrix $\tilde{\rho}_T(\tau)$; In order to evaluate it, we rely on an approximation known as the Born approximation. This approximation assumes that the environment degrees of freedom are large and thus the effect on the environment due to the system is negligibly small for a weak system-environment coupling. As a consequence, we write $\tilde{\rho}_T(\tau) = \tilde{\rho}_s(\tau) \otimes R_0 + \mathcal{O}(\tilde{H}_I)$ within the second order perturbation in system-environment interaction [34, 35, 36, 37, 38, 39, 40, 41, 42, 43]. Therefore we can write the equation (21) in time-local form as

$$\frac{d\tilde{\rho}_s(t)}{dt} = -i \text{Tr}_R[\tilde{H}_I(t), \rho_s(0) \otimes R_0]$$

$$- \int_0^t d\tau \text{Tr}_R[\tilde{H}_I(t), [\tilde{H}_I(\tau), \tilde{\rho}_s(t) \otimes R_0]]. \quad (22)$$

We note here that, for obtaining the non-Markovian time-convolutionless master equation (22), we replaced $\tilde{\rho}_s(\tau)$ with $\tilde{\rho}_s(t)$. This replacement is equivalent to obtaining a time-convolutionless master equation perturbatively up to only second order in the interaction Hamiltonian using the time-convolutionless projection operator technique [34, 39, 40]. It has been shown in a number of cases that time-local approach works better than time-nonlocal approach [34, 37, 41, 42, 44]. Now we will consider the second order time-convolutionless master equation (22) with the time variable τ replaced by $(t - \tau)$.

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & -i \text{Tr}_R[\tilde{H}_I(t), \rho_s(0) \otimes R_o] \\ & - \int_0^t d\tau \text{Tr}_R[\tilde{H}_I(t), [\tilde{H}_I(t - \tau), \tilde{\rho}_s(t) \otimes R_0]]. \end{aligned} \quad (23)$$

Next, we will study the Markovian dynamics of the system. To this end we assume that the correlation time scale τ_c for the environmental fluctuations is negligibly small compared to the relaxation time scale τ_s for the system, i.e., $\tau_c \ll \tau_s$. This time scale assumption is motivated by the condition $J^* e^{-g^2} \ll \omega$ already mentioned in section 3. The Markov approximation ($\tau_c \ll \tau_s$) allows us to set the upper limit of the integral to ∞ in equation (23). Thus we obtain the second order time-convolutionless Markovian master equation (24):

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & -i \text{Tr}_R[\tilde{H}_I(t), \rho_s(0) \otimes R_o] \\ & - \int_0^\infty d\tau \text{Tr}_R[\tilde{H}_I(t), [\tilde{H}_I(t - \tau), \tilde{\rho}_s(t) \otimes R_0]]. \end{aligned} \quad (24)$$

Defining $\{|n\rangle_{ph}\}$ as the basis set for phonons, therefore, we can write the master equation as:

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & -i \sum_n {}_{ph}\langle n | [\tilde{H}_I(t), \rho_s(0) \otimes R_o] | n \rangle_{ph} \\ & - \sum_n \int_0^\infty d\tau \left[{}_{ph}\langle n | \tilde{H}_I(t) \tilde{H}_I(t - \tau) \tilde{\rho}_s(t) \otimes R_o | n \rangle_{ph} \right. \\ & \quad - {}_{ph}\langle n | \tilde{H}_I(t) \tilde{\rho}_s(t) \otimes R_o \tilde{H}_I(t - \tau) | n \rangle_{ph} \\ & \quad - {}_{ph}\langle n | \tilde{H}_I(t - \tau) \tilde{\rho}_s(t) \otimes R_o \tilde{H}_I(t) | n \rangle_{ph} \\ & \quad \left. + {}_{ph}\langle n | \tilde{\rho}_s(t) \otimes R_o \tilde{H}_I(t - \tau) \tilde{H}_I(t) | n \rangle_{ph} \right]. \end{aligned} \quad (25)$$

In order to simplify the above master equation, we need to evaluate the time evolution of the operators involved in H_I . Considering the second term in the equation (25), yields

$$\begin{aligned} & {}_{ph}\langle n | \tilde{H}_I(t) \tilde{H}_I(t - \tau) \tilde{\rho}_s(t) \otimes R_o | n \rangle_{ph} \\ & = \sum_m e^{iH_s t} {}_{ph}\langle n | H_I | m \rangle_{ph} e^{-iH_s t} e^{iH_s(t-\tau)} {}_{ph}\langle m | H_I | n \rangle_{ph} e^{-iH_s(t-\tau)} \tilde{\rho}_s(t) \frac{e^{-\beta\omega_n}}{Z} e^{i(\omega_n - \omega_m)\tau}. \end{aligned} \quad (26)$$

In momentum space, we express HCB operators as: $b_j^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{ikr_j} b_k^\dagger$ and $b_j = \frac{1}{\sqrt{N}} \sum_k e^{-ikr_j} b_k$; then, it is important to note that the hopping term in the system Hamiltonian can be written as:

$$\begin{aligned} 0.5J \sum_{i,j>i} (e^{-g^2} b_i^\dagger b_j + \text{H.c.}) &= 0.5J e^{-g^2} \left[\sum_{i,j} b_i^\dagger b_j - \sum_i b_i^\dagger b_i \right] \\ &= 0.5J^* \left(\frac{N}{N-1} \right) e^{-g^2} \hat{n}_0 - 0.5J e^{-g^2} \hat{N}_p \\ &= \sum_k \epsilon_k b_k^\dagger b_k, \end{aligned} \quad (27)$$

where we used $J = J^*/(N-1)$, $\hat{N}_p \equiv \sum_k b_k^\dagger b_k$ and $\hat{n}_0 \equiv b_0^\dagger b_0$ (i.e., the particle number in momentum $k=0$ state). Here it should be mentioned that using HCBs instead of spins has enabled us to obtain (with ease) the excitation spectrum ϵ_k which is crucial for the analysis given below. Let $\{|q\rangle_s\}$ denote the complete set of energy eigenstates (with eigenenergies E_q^s) of the system Hamiltonian H_s ; then we can write:

$$\begin{aligned} e^{iH_s t} H_I e^{-iH_s t} &= 0.5J e^{-g^2} \sum_{l,j>l} \sum_{q,q'} |q\rangle_{ss} \langle q| e^{iH_s t} \left[\frac{1}{N} \sum_{k,p} b_k^\dagger b_p e^{i(kr_l - pr_j)} \right] e^{-iH_s t} |q'\rangle_{ss} \langle q'| \{ \mathcal{S}_+^{lj\dagger} \mathcal{S}_-^{lj} - 1 \} \\ &\quad + \text{H.c.}, \end{aligned} \quad (28)$$

which implies

$$e^{iH_s t} {}_{ph} \langle n | H_I | m \rangle_{ph} e^{-iH_s t} = \sum_{q,q'} |q\rangle_{ss} \langle q| {}_{ph} \langle n | H_I | m \rangle_{ph} |q'\rangle_{ss} \langle q'| e^{i(E_q^s - E_{q'}^s)t}, \quad (29)$$

where $|E_q^s - E_{q'}^s| = 0.5J^* \left(\frac{N}{N-1} \right) e^{-g^2}$ or 0. Here we have taken the total number of HCBs to be conserved; then, only the hopping term in H_s will contribute to the particle excitation energy. Substituting equation (29) in equation (26), we get

$$\begin{aligned} &{}_{ph} \langle n | \tilde{H}_I(t) \tilde{H}_I(t-\tau) \tilde{\rho}_s(t) \otimes R_o | n \rangle_{ph} \\ &= \sum_m \sum_{q,q',q''} \left[|q\rangle_{ss} \langle q| {}_{ph} \langle n | H_I | m \rangle_{ph} |q'\rangle_{ss} \langle q'| {}_{ph} \langle m | H_I | n \rangle_{ph} |q''\rangle_{ss} \langle q''| e^{i[(E_q^s - E_{q'}^s)t + (E_{q'}^s - E_{q''}^s)(t-\tau)]} \right] \\ &\quad \times \tilde{\rho}_s(t) \frac{e^{-\beta\omega_n}}{Z} e^{i(\omega_n - \omega_m)\tau}. \end{aligned} \quad (30)$$

Thus under the assumption of $J^* e^{-g^2} \ll \omega$, it follows that $|\omega_n - \omega_m| \gg |E_q^s - E_{q'}^s|$ and $|\omega_n - \omega_m| \gg |E_{q'}^s - E_{q''}^s|$; hence in equation (30), we can take $e^{i[(E_q^s - E_{q'}^s)t]} = 1$ and $e^{i[(E_{q'}^s - E_{q''}^s)(t-\tau)]} = 1$ which implies that we do not get terms producing decay. The resultant equation is

$${}_{ph} \langle n | \tilde{H}_I(t) \tilde{H}_I(t-\tau) \tilde{\rho}_s(t) \otimes R_o | n \rangle_{ph} = \sum_m {}_{ph} \langle n | H_I | m \rangle_{ph} {}_{ph} \langle m | H_I | n \rangle_{ph} \tilde{\rho}_s(t) \frac{e^{-\beta\omega_n}}{Z} e^{i(\omega_n - \omega_m)\tau}. \quad (31)$$

Carrying out the same analysis on the remaining (i.e., third, fourth, and fifth) terms in the master equation, we write equation (25) as:

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & -i \sum_n {}_{ph}\langle n | [\tilde{H}_I(t), \tilde{\rho}_s(0) \otimes R_o] | n \rangle_{ph} \\ & - \sum_{n,m} \int_0^\infty d\tau \left[{}_{ph}\langle n | H_I | m \rangle_{ph} {}_{ph}\langle m | H_I | n \rangle_{ph} \tilde{\rho}_s(t) \frac{e^{-\beta\omega_n}}{Z} e^{i(\omega_n - \omega_m)\tau} \right. \\ & - {}_{ph}\langle n | H_I | m \rangle_{ph} \tilde{\rho}_s(t) {}_{ph}\langle m | H_I | n \rangle_{ph} \frac{e^{-\beta\omega_m}}{Z} e^{i(\omega_n - \omega_m)\tau} \\ & - {}_{ph}\langle n | H_I | m \rangle_{ph} \tilde{\rho}_s(t) {}_{ph}\langle m | H_I | n \rangle_{ph} \frac{e^{-\beta\omega_m}}{Z} e^{-i(\omega_n - \omega_m)\tau} \\ & \left. + \tilde{\rho}_s(t) {}_{ph}\langle n | H_I | m \rangle_{ph} {}_{ph}\langle m | H_I | n \rangle_{ph} \frac{e^{-\beta\omega_n}}{Z} e^{-i(\omega_n - \omega_m)\tau} \right]. \quad (32) \end{aligned}$$

Next, we evaluate the first term in the above equation and show that it is zero at $T = 0$. We observe that

$$\begin{aligned} Tr_R[\tilde{H}_I(t)R_o] &= \sum_n {}_{ph}\langle n | \tilde{H}_I(t)R_o | n \rangle_{ph} \\ &= 0.5J e^{-g^2} \sum_{l,j \neq l} \left[e^{iH_s t} b_l^\dagger b_j e^{-iH_s t} {}_{ph}\langle 0 | \{ \mathcal{S}_+^{lj\dagger} \mathcal{S}_-^{lj} - 1 \} | 0 \rangle_{ph} \right] \\ &= 0. \end{aligned} \quad (33)$$

Thus, we have $\sum_n {}_{ph}\langle n | [\tilde{H}_I(t), \rho_s(0) \otimes R_o] | n \rangle_{ph} = 0$ and the master equation at $T = 0$ simplifies as:

$$\begin{aligned} \frac{d\tilde{\rho}_s(t)}{dt} = & - \sum_m \int_0^\infty d\tau \left[|{}_{ph}\langle 0 | H_I | m \rangle_{ph}|^2 \tilde{\rho}_s(t) e^{-i\omega_m \tau} + \tilde{\rho}_s(t) |{}_{ph}\langle 0 | H_I | m \rangle_{ph}|^2 e^{i\omega_m \tau} \right] \\ & + \sum_n \int_0^\infty d\tau \left[{}_{ph}\langle n | H_I | 0 \rangle_{ph} \tilde{\rho}_s(t) {}_{ph}\langle 0 | H_I | n \rangle_{ph} e^{i\omega_n \tau} \right. \\ & \quad \left. + {}_{ph}\langle n | H_I | 0 \rangle_{ph} \tilde{\rho}_s(t) {}_{ph}\langle 0 | H_I | n \rangle_{ph} e^{-i\omega_n \tau} \right] \\ = & - \sum_n \left[\int_0^\infty d\tau e^{-i(\omega_n - i\eta)\tau} |{}_{ph}\langle 0 | H_I | n \rangle_{ph}|^2 \tilde{\rho}_s(t) \right. \\ & + \int_0^\infty d\tau e^{i(\omega_n + i\eta)\tau} \tilde{\rho}_s(t) |{}_{ph}\langle 0 | H_I | n \rangle_{ph}|^2 \\ & \left. - \int_{-\infty}^\infty d\tau e^{i\omega_n \tau} {}_{ph}\langle n | H_I | 0 \rangle_{ph} \tilde{\rho}_s(t) {}_{ph}\langle 0 | H_I | n \rangle_{ph} \right]. \quad (34) \end{aligned}$$

Now, we know $\int_{-\infty}^\infty d\tau e^{i\omega_n \tau} \propto \delta(\omega_n)$. Therefore, on using this relation and the fact that ${}_{ph}\langle 0 | H_I | 0 \rangle_{ph} = 0$, the third term in equation (34) vanishes; hence, we get

$$\frac{d\tilde{\rho}_s(t)}{dt} = i \sum_n \left[\frac{|{}_{ph}\langle 0 | H_I | n \rangle_{ph}|^2}{\omega_n} \tilde{\rho}_s(t) - \tilde{\rho}_s(t) \frac{|{}_{ph}\langle 0 | H_I | n \rangle_{ph}|^2}{\omega_n} \right]. \quad (35)$$

The term $\sum_n \left[\frac{|{}_{ph}\langle 0 | H_I | n \rangle_{ph}|^2}{\omega_n} \right]$ corresponds to the effective Hamiltonian $H^{(2)}$ in second order perturbation and commutes with H_0 (see section 3). Let $|n\rangle_s$ be the simultaneous

eigenstate for $H^{(2)}$ and H_s with eigenvalues $E_n^{(2)}$ and E_n^s , respectively. Then, from the above equation we get:

$${}_s\langle n|\tilde{\rho}_s(t)|m\rangle_s = e^{-i(E_n^{(2)} - E_m^{(2)})t} {}_s\langle n|\tilde{\rho}_s(0)|m\rangle_s, \quad (36)$$

which implies that

$${}_s\langle n|\rho_s(t)|m\rangle_s = e^{-i(E_n - E_m)t} {}_s\langle n|\rho_s(0)|m\rangle_s, \quad (37)$$

where $E_n = E_n^s + E_n^{(2)}$. Thus we see from the above equation that there is only a phase shift but no decoherence! Since the matrix elements of an operator are invariant under canonical transformation, it should be clear that no loss in off-diagonal density matrix elements (i.e., no decoherence) in the LF transformed frame of reference implies no loss in off-diagonal density matrix elements (i.e., no decoherence) in the original untransformed frame of reference. Although the HCB's in the original frame of reference form polarons and are thus entangled with the environment, nevertheless no decoherence results. For greater clarity, the form of ${}_s\langle n|\rho_s(t)|m\rangle_s$ in the original frame of reference and its associated non-decoherence is discussed in Appendix B for a special two-spin case of IRHM. Thus, up to second order in perturbation, the assumption $J^*e^{-g^2} \ll \omega$, the infinite range of the Heisenberg model, and the Markov approximation ($\tau_c \ll \tau_s$) together have ensured that the system, with a fixed S_T^z , does not decohere.

While the above analysis is valid in the regime $k_B T/\omega \ll 1$, the finite temperature case $k_B T/\omega \gtrsim 1$ needs additional extensive considerations and will be dealt with elsewhere [45].

4.2. Decoherence due to Global Phonons:

We will now analyze decoherence due to interaction of the spin system with global phonons. To this end, we consider the following total Hamiltonian where all qubits of our IRHM interact identically with the environment.

$$\begin{aligned} H_{Tot} = H_s + \sum_i S_i^z \sum_k \omega_k (g_k a_k^\dagger + g_k^* a_k) \\ + \sum_k \omega_k a_k^\dagger a_k, \end{aligned} \quad (38)$$

where $H_s = H_{\text{IRHM}}$ is the Hamiltonian of the system. (Here, for global phonons, since we do not use LF transformation, we define H_s as the untransformed system Hamiltonian.) Since the z-component of the total spin S_{Total}^z (and thus the interaction Hamiltonian) commutes with H_{IRHM} , the eigenstates having same eigenvalue S_T^z constitute a DFS. To study the case when S_T^z is not conserved and to obtain the form of the reduced density matrix $\rho_s(t)$, we study the dynamics of the system through the following non-Markovian master equation [46]:

$$\begin{aligned} \frac{d\rho_s(t)}{dt} = & -i[H_s, \rho_s(t)] + F(t)[L\rho_s(t), L] \\ & + F^*(t)[L, \rho_s(t)L], \end{aligned} \quad (39)$$

where L is the system operator that couples with the bath and satisfies the constraint $[L, H_s] = 0$. For the total Hamiltonian in equation (38), $L = \sum_i S_i^z = S_{Total}^z$. Also, $F(t) = \int_0^t \alpha(t-s)ds$ where $\alpha(t-s) = \eta(t-s) + i\nu(t-s)$ is the bath correlation function at temperature T with

$$\begin{aligned}\eta(t-s) &= \sum_k |g_k|^2 \omega_k^2 \coth\left(\frac{\omega_k}{2k_B T}\right) \cos[\omega_k(t-s)], \\ \nu(t-s) &= - \sum_k |g_k|^2 \omega_k^2 \sin[\omega_k(t-s)].\end{aligned}\tag{40}$$

The function $F(t)$ governs the non-Markovian dynamical features of the system.

Let $\{|n\rangle_s\}$ be the eigen basis in which both the operators S_{Total}^z and H_s are simultaneously diagonalized. Upon solving the master equation explicitly we get [46]:

$$\begin{aligned}{}_s\langle n|\rho_s(t)|m\rangle_s &= \exp\left(-i\left[(E_n^s - E_m^s)t + \{(S_{Tn}^z)^2 - (S_{Tm}^z)^2\}Y(t)\right]\right) \\ &\times \exp\left[-(S_{Tn}^z - S_{Tm}^z)^2 X(t)\right] {}_s\langle n|\rho_s(0)|m\rangle_s,\end{aligned}\tag{41}$$

where E_n^s and S_{Tn}^z are defined through $H_s|n\rangle_s = E_n^s|n\rangle_s$ and $\sum_i S_i^z|n\rangle_s = S_{Tn}^z|n\rangle_s$. Furthermore, $X(t) \equiv \int_0^t F_R(s)ds$ and $Y(t) \equiv \int_0^t F_I(s)ds$ with $F_R(t) + iF_I(t) \equiv F(t)$. This implies that, when states $|m\rangle_s$ and $|n\rangle_s$ have the same z-component of the total spin S_T^z (i.e., $S_{Tn}^z = S_{Tm}^z$), the matrix elements ${}_s\langle n|\rho_s(t)|m\rangle_s = {}_s\langle n|\rho_s(0)|m\rangle_s \exp[-i(E_n^s - E_m^s)t]$ display a decoherence free behaviour. But when S_T^z is not conserved, the off-diagonal matrix elements will diminish in general, i.e., the system undergoes decoherence. In the language of HCBs, the eigenstates of the system with a fixed number of HCBs makeup a DFS. Furthermore, the entanglement entropy of the system will remain unaltered since the density matrix evolves unitarily. In future, using the above framework, we will consider the interesting case of dynamical evolution and decoherence of states with different S_T^z values.

5. Discussion and Conclusions

In conclusion, we have shown that the eigenstates of H_{eff} are the same as those of H_{IRHM} and for Markov processes they are decoherence free under the coupling of the system to local optical phonons. For global optical phonons (i.e., when all the qubits are exposed to the same collective noise) the eigenstates with the same S_T^z form a DFS. A DFS is expected in the global phonon case because the Hamiltonian of the system commutes with $\sum_i S_i^z$. But the important point is that, even in the local phonon case, it is still possible to fully preserve coherence for the composite particle (i.e., polaronic HCB) system with a fixed number of particles. More specifically, for local phonons, ${}_s\langle n|\rho_s(t)|m\rangle_s$ differs from ${}_s\langle n|\rho_s(0)|m\rangle_s$ only by a phase factor and ${}_s\langle n|\rho_s(0)|m\rangle_s$ can be obtained from ${}_s\langle n|\rho_{IRHM}|m\rangle_s$ (density matrix element of IRHM) by an exact unitary evolution [24]. Later, we will analyze the non-Markov processes and see how the resultant dynamics deviates from the Markovian dynamics.

Earlier, a new type of resonating valence bond (RVB) states [47] were constructed for four and six spins using homogenized linear superposition of the $S_T = 0$ states of H_{IRHM} ; these RVB states have a high bipartite entanglement. The decoherence analysis in this paper is also applicable to these new RVB states which are groundstates of our H_{IRHM} . Our RVB states are constructed using valence bond (VB) states which are $S_T = 0$ states. VB states are built from singlet states between pairs of spins. A general VB state is defined as:

$$|\Psi\rangle_{\text{vb}} = |\Phi_{i_1, j_1}\rangle \otimes |\Phi_{i_2, j_2}\rangle \otimes \dots \otimes |\Phi_{i_M, j_M}\rangle,$$

where $|\Phi_{i_k, j_k}\rangle \equiv \frac{1}{\sqrt{2}} (|\frac{1}{2}\rangle_{i_k} |-\frac{1}{2}\rangle_{j_k} - |-\frac{1}{2}\rangle_{i_k} |\frac{1}{2}\rangle_{j_k})$ denotes the singlet dimer connecting a site i_k in sub-lattice A with a site j_k in sub-lattice B . Examples of our RVB states (that are constructed from spins 1, 2, 3, 4, ... arranged sequentially on the vertices of a regular polygon and that have high bipartite entanglement) are $|\Psi_4\rangle_{\text{rvb}}$ given below for four spins:

$$|\Psi_4\rangle_{\text{rvb}} \equiv \omega_3 (|\Phi_{1,2}\rangle \otimes |\Phi_{3,4}\rangle) + \omega_3^2 (|\Phi_{1,4}\rangle \otimes |\Phi_{2,3}\rangle), \quad (42)$$

where $\omega_3 (= e^{i2\pi/3})$ is a cube root of unity; and $|\Psi_6\rangle_{\text{rvb}}$ given below for six spins:

$$\begin{aligned} |\Psi_6\rangle_{\text{rvb}} = & \omega_4 (|\Phi_{1,2}\rangle \otimes |\Phi_{3,6}\rangle \otimes |\Phi_{4,5}\rangle) \\ & + \omega_4^2 (|\Phi_{2,3}\rangle \otimes |\Phi_{1,4}\rangle \otimes |\Phi_{5,6}\rangle) \\ & + \omega_4^3 (|\Phi_{1,6}\rangle \otimes |\Phi_{2,5}\rangle \otimes |\Phi_{3,4}\rangle), \end{aligned} \quad (43)$$

where $\omega_4 (= e^{i2\pi/4})$ is a fourth root of unity.

Before closing we will make a few general remarks. Firstly, the H_{IRHM} model of this paper, deals with the extreme case of distance independent interaction among the spins. On the other extreme end, if one were to consider a nearest-neighbor interaction anisotropic Heisenberg chain [of the type $\sum_i \{J_\perp (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_\parallel S_i^z S_{i+1}^z\}$ where $J_\perp > J_\parallel > 0$] with a strong coupling to local phonons [introducing the additional terms $g\omega \sum_i S_i^z (a_i^\dagger + a_i) + \omega \sum_i a_i^\dagger a_i$], then the spin system undergoes a Luttinger liquid to a spin-density-wave transition upon turning on the spin-phonon interaction and decoheres [48]. In general, distance-dependent-interaction in spin Hamiltonians will fall somewhere in between the above two extreme cases.

Next, our decoherence analysis for local optical phonons will continue to be valid even for the more general optical phonon terms given below:

$$\frac{1}{N^{1/2}} \sum_{i,k} S_i^z [\omega_k (g_k a_{k,i}^\dagger + g_k^* a_{k,i})] + \sum_{k,i} \omega_k a_{k,i}^\dagger a_{k,i}. \quad (44)$$

We also must mention that our approach cannot accommodate the acoustic phonon case as here the condition $J^* e^{-g^2} \ll \omega_k$ cannot be satisfied in the long wavelength limit.

Acknowledgments

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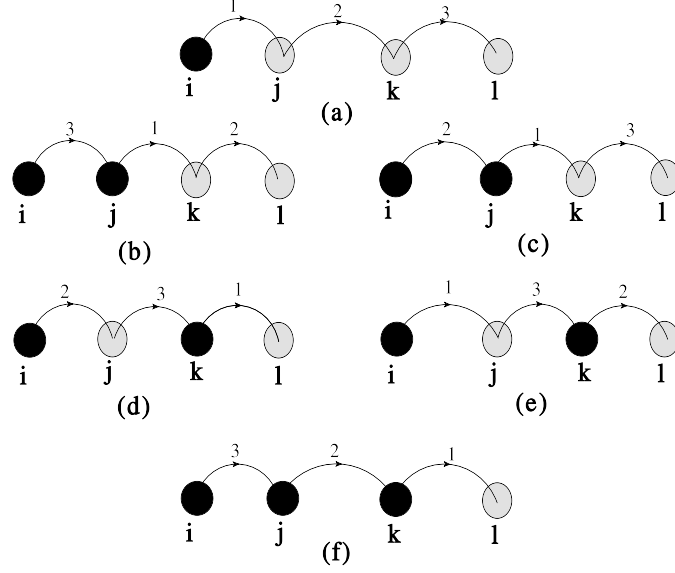


Figure A1. Open loop hopping processes contributing to effective hopping term T_n^{li} in third-order perturbation theory. Here empty circles correspond to sites with no particles while filled circles correspond to sites with hard-core-bosons. The numbers 1, 2, and 3 indicate the order of hopping.

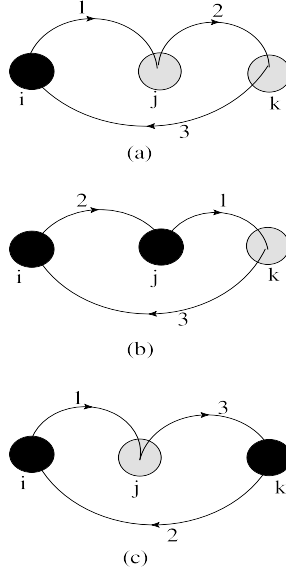


Figure A2. Closed-loop hopping processes contributing to effective interaction term V_n^i in third-order perturbation theory. Here filled (empty) circles correspond to sites with (without) hard-core-bosons. The numbers 1, 2, and 3 represent hopping sequence.

Appendix A.

In this appendix, we will show that the third-order perturbation theory also produces a term that has the same eigenstates as IRHM. To this end, we obtain the following

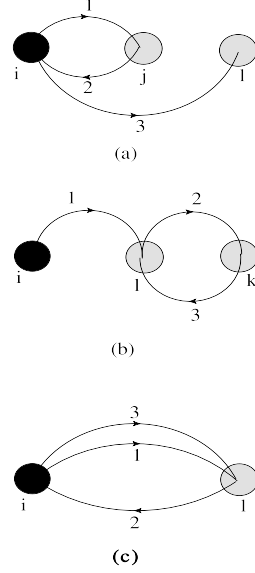


Figure A3. Hopping processes (involving closed loops) contributing to effective hopping term $T_{C_n}^{li}$ in third-order perturbation theory. Filled (empty) circles represent occupied (unoccupied) sites.

third-order perturbation term in the effective Hamiltonian:

$$H^{(3)} = \sum_{m \neq 0, n \neq 0} \frac{{}_{ph}\langle 0 | H_I | m \rangle_{ph} {}_{ph}\langle m | H_I | n \rangle_{ph} {}_{ph}\langle n | H_I | 0 \rangle_{ph}}{\Delta E_m^{ph} \Delta E_n^{ph}}. \quad (\text{A.1})$$

Here $\Delta E_m^{ph} = \omega_m - \omega_0$. Evaluation of $H^{(3)}$ leads to various hopping terms and interaction terms.

$$H^{(3)} = \sum_{i, l \neq i} \left[\sum_{n=1}^6 t_n T_n^{li} + \sum_{n=1}^3 t_{cn} T_{C_n}^{li} \right] + \sum_i \sum_{n=1}^3 v_n V_n^i, \quad (\text{A.2})$$

where $t_n \sim (J^3 e^{-g^2}) / (g^2 \omega)^2$, $t_{cn} \sim J^3 e^{-g^2} / (g \omega)^2$, and $v_n \sim J^3 / (g^2 \omega)^2$ (as will be explained later). We will demonstrate below that $H^{(3)}$ is of the following form

$$H^{(3)} = \sum_{i, l > i} \left[T \left(\sum_k n_k \right) b_l^\dagger b_i + \text{H.c.} \right] + \sum_i V \left(\sum_k n_k \right) n_i, \quad (\text{A.3})$$

where T and V are functions of the total number operator $\sum_k n_k$. Since the IRHM commutes with the total number operator, $H^{(3)}$ has the same eigenstates as IRHM!

There are six open-loop hopping processes T_n^{li} depicted in figure A1. We analyze them sequentially below.

$$T_1^{li} = \sum_{k \neq i, l, j} \sum_{j \neq i, l} b_l^\dagger b_k b_k^\dagger b_j b_j^\dagger b_i$$

$$\begin{aligned}
&= \sum_{k \neq i, l, j} (1 - b_k^\dagger b_k) \sum_{j \neq i, l} (1 - b_j^\dagger b_j) b_l^\dagger b_i \\
&= \left[\sum_{k \neq i, l} (1 - b_k^\dagger b_k) - 1 \right] \left[\sum_{j \neq i, l} (1 - b_j^\dagger b_j) \right] b_l^\dagger b_i \\
&= \left[\sum_{k \neq i, l} (1 - b_k^\dagger b_k) - 1 \right] \left[(N - 2) - \sum_{j \neq l} b_j^\dagger b_j \right] b_l^\dagger b_i \\
&= \left[\sum_{k \neq i, l} (1 - b_k^\dagger b_k) - 1 \right] \left[(N - 1) - \sum_j b_j^\dagger b_j \right] b_l^\dagger b_i \\
&= \left[(N - 1) - \sum_j b_j^\dagger b_j \right] \left[\sum_{k \neq i, l} (1 - b_k^\dagger b_k) - 1 \right] b_l^\dagger b_i \\
&= \left[(N - 1) - \sum_j b_j^\dagger b_j \right] \left[(N - 2) - \sum_k b_k^\dagger b_k \right] b_l^\dagger b_i.
\end{aligned} \tag{A.4}$$

The second hopping process T_2^{li} in figure A1 (b) is given by

$$\begin{aligned}
T_2^{li} &= \sum_{k \neq i, l, j} \sum_{j \neq i, l} b_j^\dagger b_i b_l^\dagger b_k b_k^\dagger b_j \\
&= \sum_{k \neq i, l, j} (1 - b_k^\dagger b_k) \sum_{j \neq i, l} b_j^\dagger b_j b_l^\dagger b_i \\
&= \sum_{k \neq i, l} (1 - b_k^\dagger b_k) \sum_{j \neq i, l} b_j^\dagger b_j b_l^\dagger b_i \\
&= \sum_{k \neq i, l} (1 - b_k^\dagger b_k) \left[\sum_j b_j^\dagger b_j - 1 \right] b_l^\dagger b_i \\
&= \left[\sum_j b_j^\dagger b_j - 1 \right] \left[(N - 1) - \sum_k b_k^\dagger b_k \right] b_l^\dagger b_i.
\end{aligned} \tag{A.5}$$

The hopping process T_3^{li} in figure A1 (c) is expressed as $T_3^{li} = \sum_{k \neq i, l, j} \sum_{j \neq i, l} b_l^\dagger b_k b_k^\dagger b_i b_k^\dagger b_j = T_2^{li}$. The fourth hopping process T_4^{li} in figure A1 (d) is obtained as follows.

$$\begin{aligned}
T_4^{li} &= \sum_{j \neq i, l, k} \sum_{k \neq i, l} b_k^\dagger b_j b_j^\dagger b_i b_l^\dagger b_k \\
&= \sum_{j \neq i, l, k} (1 - b_j^\dagger b_j) \sum_{k \neq i, l} b_k^\dagger b_k b_l^\dagger b_i \\
&= T_2^{li}.
\end{aligned} \tag{A.6}$$

The hopping process T_5^{li} in figure A1 (e) yields $T_5^{li} = \sum_{j \neq i, l, k} \sum_{k \neq i, l} b_k^\dagger b_j b_l^\dagger b_k b_j^\dagger b_i = T_4^{li}$. We analyze below the last hopping process T_6^{li} in figure A1 (f).

$$T_6^{li} = \sum_{k \neq i, l, j} \sum_{j \neq i, l} b_j^\dagger b_i b_k^\dagger b_j b_l^\dagger b_k$$

$$\begin{aligned}
&= \sum_{k \neq i, l, j} b_k^\dagger b_k \sum_{j \neq i, l} b_j^\dagger b_j b_l^\dagger b_i \\
&= \left[\sum_{k \neq i, l} b_k^\dagger b_k - 1 \right] \sum_{j \neq i, l} b_j^\dagger b_j b_l^\dagger b_i \\
&= \left[\sum_{k \neq i, l} b_k^\dagger b_k - 1 \right] \left[\sum_j b_j^\dagger b_j - 1 \right] b_l^\dagger b_i \\
&= \left[\sum_j b_j^\dagger b_j - 1 \right] \left[\sum_k b_k^\dagger b_k - 2 \right] b_l^\dagger b_i.
\end{aligned} \tag{A.7}$$

We will now deal with closed-loop hopping processes such as those in figure A2. These lead to effective interactions. The process V_1^i in figure A2 (a), obtained from figure A1 (a) by setting $l = i$, is given as follows.

$$\begin{aligned}
V_1^i &= \sum_{k \neq i, j} \sum_{j \neq i} b_i^\dagger b_k b_k^\dagger b_j b_j^\dagger b_i \\
&= \sum_{k \neq i, j} (1 - b_k^\dagger b_k) \sum_{j \neq i} (1 - b_j^\dagger b_j) b_i^\dagger b_i \\
&= \left[\sum_{k \neq i} (1 - b_k^\dagger b_k) - 1 \right] \left[\sum_{j \neq i} (1 - b_j^\dagger b_j) \right] b_i^\dagger b_i \\
&= \left[(N) - \sum_j b_j^\dagger b_j \right] \left[(N - 1) - \sum_k b_k^\dagger b_k \right] b_i^\dagger b_i.
\end{aligned} \tag{A.8}$$

Next, the hopping process V_2^i corresponding to closed loop in figure A2 (b) is obtained from figure A1 (c) by taking $l = i$.

$$\begin{aligned}
V_2^i &= \sum_{k \neq i, j} \sum_{j \neq i} b_i^\dagger b_k b_j^\dagger b_i b_k^\dagger b_j \\
&= \sum_{k \neq i, j} (1 - b_k^\dagger b_k) \sum_{j \neq i} b_j^\dagger b_j b_i^\dagger b_i \\
&= \sum_{k \neq i} (1 - b_k^\dagger b_k) \sum_{j \neq i} b_j^\dagger b_j b_i^\dagger b_i \\
&= \sum_{k \neq i} (1 - b_k^\dagger b_k) \left[\sum_j b_j^\dagger b_j - 1 \right] b_i^\dagger b_i \\
&= \left[\sum_j b_j^\dagger b_j - 1 \right] \left[(N) - \sum_k b_k^\dagger b_k \right] b_i^\dagger b_i.
\end{aligned} \tag{A.9}$$

Lastly, the hopping V_3^i [depicted by the closed loop in figure A2 (c)] is obtained from figure A1 (e) by setting $l = i$.

$$\begin{aligned}
V_3^i &= \sum_{j \neq i, k} \sum_{k \neq i} b_k^\dagger b_j b_i^\dagger b_k b_j^\dagger b_i \\
&= \sum_{j \neq i, k} (1 - b_j^\dagger b_j) \sum_{k \neq i} b_k^\dagger b_k b_i^\dagger b_i
\end{aligned}$$

$$= V_2^i. \quad (\text{A.10})$$

Finally, we consider figures A3 (a), (b), and (c) which deal with effective hopping terms T_{Cn}^{li} involving closed loops. The effective hopping term T_{C1}^{li} , corresponding to figure A3 (a), is obtained by setting $k = i$ in figure A1 (a):

$$\begin{aligned} T_{C1}^{li} &= \sum_{j \neq i, l} b_l^\dagger b_i b_i^\dagger b_j b_j^\dagger b_i \\ &= \sum_{j \neq i, l} (1 - b_j^\dagger b_j) b_l^\dagger b_i \\ &= \left[(N - 2) - \sum_{j \neq l} b_j^\dagger b_j \right] b_l^\dagger b_i \\ &= \left[(N - 1) - \sum_j b_j^\dagger b_j \right] b_l^\dagger b_i. \end{aligned} \quad (\text{A.11})$$

To obtain the effective hopping term T_{C2}^{li} corresponding to figure A3 (b), we take $j = l$ in figure A1 (a):

$$\begin{aligned} T_{C2}^{li} &= \sum_{k \neq i, l} b_l^\dagger b_k b_k^\dagger b_l b_l^\dagger b_i \\ &= \sum_{k \neq i, l} (1 - b_k^\dagger b_k) b_l^\dagger b_i \\ &= \left[(N - 2) - \sum_{k \neq l} b_k^\dagger b_k \right] b_l^\dagger b_i \\ &= \left[(N - 1) - \sum_k b_k^\dagger b_k \right] b_l^\dagger b_i \\ &= T_{C1}^{li}. \end{aligned} \quad (\text{A.12})$$

The effective hopping term T_{C3}^{li} depicted in figure A3 (c) [upon setting $k = i$ and $j = l$ in figure A1 (a)] is given by

$$T_{C3}^{li} = b_l^\dagger b_i b_i^\dagger b_l b_l^\dagger b_i = b_l^\dagger b_i. \quad (\text{A.13})$$

Thus we have shown that $H^{(3)}$ contains effective hopping terms $\sum_{i, l > i} [T(\sum_k n_k) b_l^\dagger b_i + \text{H.c.}]$ and effective interaction terms $(\sum_i V(\sum_k n_k) n_i)$. Since T and V are functions of the total number operator, $H^{(3)}$ and IRHM have the same eigenstates. These arguments can be extended to even higher-order perturbation theory to show that the effective Hamiltonian (after taking all orders of perturbation into account) will give the same eigenstates as IRHM!

We will now explain the expressions for the coefficients t_n , v_n , and t_{cn} in equation (A.2), obtained from third-order perturbation theory, using typical schematic diagrams shown in figure A4 [for details of corresponding diagrams and analysis in second order perturbation, see reference [26]]. We consider two distinct time scales associated with hopping processes between two sites: (i) $\sim 1/(Je^{-g^2})$ corresponding to either full

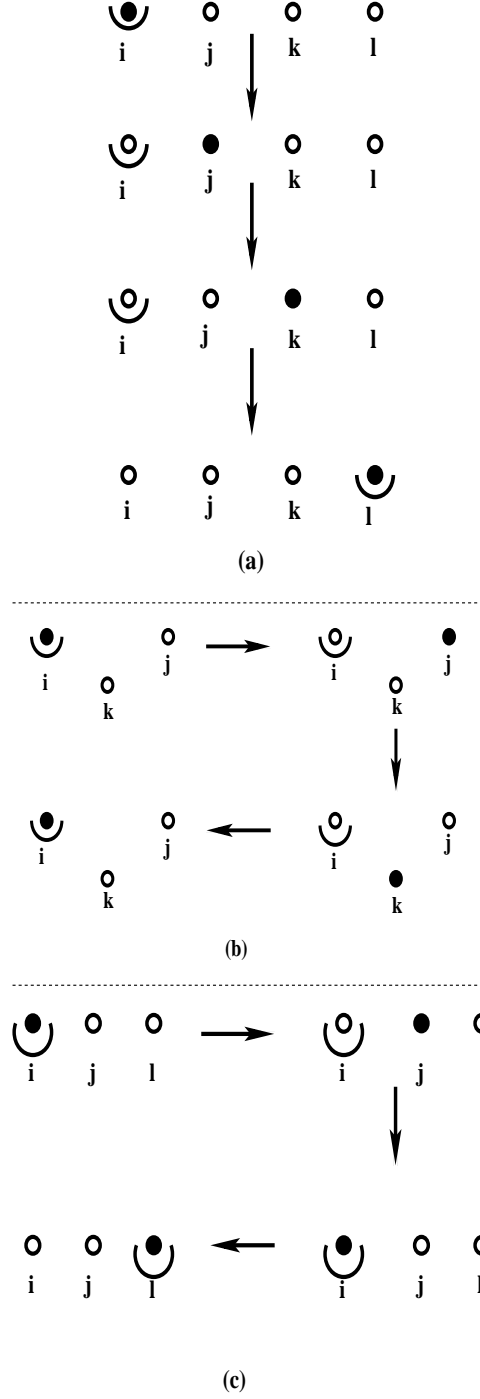


Figure A4. Schematic diagrams (a), (b), and (c), corresponding to the hopping processes depicted in figure A1 (a), figure A2 (a), and figure A3 (a), respectively, yield coefficients t_n , v_n , and t_{cn} , respectively. The intermediate states give the typical dominant contributions. Here empty circles correspond to empty sites, while filled circles indicate particle positions. Parabolic curve at a site depicts full distortion at that site with corresponding energy $-g^2\omega$ ($+g^2\omega$) if the hard-core-boson is present (absent) at that site.

distortion at a site to form a small polaronic potential well (of energy $-g^2\omega$) or full relaxation from the small polaronic distortion and (ii) $\sim 1/J$ related to negligible distortion/relaxation at a site. The coefficient t_n corresponds to the typical dominant distortion processes shown schematically in figure A4 (a) with the pertinent typical hopping processes being depicted in figure A1 (a). In figure A4 (a), after the HCB hops away from the initial site, the intermediate states have the same distortion as the initial state. Next, when the HCB hops to its final site there is a distortion at this final site with a concomitant relaxation at the initial site. Hence the contribution to the coefficient t_n becomes $J/(2g^2\omega) \times J/(2g^2\omega) \times Je^{-g^2} \sim J^3e^{-g^2}/(g^2\omega)^2$. As regards coefficient v_n , it can be deduced based on the typical dominant hopping-cum-distortion processes depicted in figure A4 (b) which typifies the hopping processes in figure A2 (a). In figure A4 (b), when the particle hops to different sites and reaches finally the initial site, there is no change in distortion at any site. Hence v_n can be estimated to be $J/(2g^2\omega) \times J/(2g^2\omega) \times J \sim J^3/(g^2\omega)^2$. Lastly, we obtain the coefficient t_{cn} by considering the typical dominant diagram in figure A4 (c) corresponding to the typical process in figure A3 (a). In figure A4 (c), where the first intermediate state depicts the particle hopping but leaving the distortion unchanged, we get a contribution $J/(2g^2\omega)$; for the next intermediate state, where the HCB returns to the initial site, the initial site has to undergo a slight relaxation (involving absorbing a phonon so as to yield a non-zero denominator in the perturbation theory) leading to the contribution J/ω ; and lastly, when the HCB hops to the final site, there is a distortion at the final site with a simultaneous relaxation at the initial site thereby producing a contribution Je^{-g^2} . Thus we calculate t_{cn} to be $J/(2g^2\omega) \times J/\omega \times Je^{-g^2} \sim J^3e^{-g^2}/(g\omega)^2$ [30].

Appendix B.

In equation (37), the matrix element ${}_s\langle n|\rho_s(t)|m\rangle_s$ can be written as

$$\begin{aligned} {}_s\langle n|\rho_s(t)|m\rangle_s &= {}_s\langle n|\left[\sum_n {}_{ph}\langle n|\rho_T(t)|n\rangle_{ph}\right]|m\rangle_s \\ &= {}_s\langle n|\sum_n {}_{ph}\langle n|e^S\rho_T^0(t)e^{-S}|n\rangle_{ph}|m\rangle_s, \end{aligned} \quad (\text{B.1})$$

where $\rho_T^0(t)$ is the total density matrix in the original frame of reference. Now, we illustrate this quantity by considering the simple two-spin (i.e., $N=2$) case of the IRHM. The singlet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ and the triplet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ are the $S_T^z = 0$ eigenstates of the two-qubit IRHM Hamiltonian; in HCB language, these states are expressed as $\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$ and $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$, respectively. Now, the operator e^{-S} can be expressed as

$$e^{-S} = e^{g\sum_{i=1,2}(n_i - \frac{1}{2})(a_i - a_i^\dagger)} = \prod_{i=1,2} e^{g(n_i - \frac{1}{2})(a_i - a_i^\dagger)} = \prod_{i=1,2} \left[n_i X_i + (1 - n_i) X_i^\dagger \right], \quad (\text{B.2})$$

where $X_i = e^{\frac{g}{2}(a_i - a_i^\dagger)}$. Using the above, we obtain

$$e^{-S} \frac{1}{\sqrt{2}} (|10\rangle \pm |01\rangle) |m_1, m_2\rangle_{ph} = [X_1 X_2^\dagger |10\rangle \pm X_2 X_1^\dagger |01\rangle] |m_1, m_2\rangle_{ph}. \quad (\text{B.3})$$

where m_1 and m_2 correspond to phonon occupation numbers at site 1 and site 2 respectively. Therefore, from equation (B.1) we can write the density matrix element between singlet and triplet states in the original frame of reference as

$$\begin{aligned} & \frac{1}{2} (\langle 10| - \langle 01|) \rho_s(t) (|10\rangle + |01\rangle) \\ &= \frac{1}{2} \sum_{m_1, m_2} {}_{ph} \langle m_1, m_2 | \left(\langle 10| X_2 X_1^\dagger - \langle 01| X_1 X_2^\dagger \right) \rho_T^o(t) \left(X_1 X_2^\dagger |10\rangle + X_2 X_1^\dagger |01\rangle \right) |m_1, m_2\rangle_{ph}. \end{aligned} \quad (\text{B.4})$$

$$(\text{B.5})$$

Depending upon the presence or absence of HCB, appropriate deformation will be produced at each site and $\left[\left(X_1 X_2^\dagger |10\rangle \pm X_2 X_1^\dagger |01\rangle \right) |m_1, m_2\rangle_{ph} \right]$ represents polaronic states. Furthermore, in equation (B.5), no loss in the off-diagonal matrix element on the left hand side implies no loss in the off-diagonal matrix element on the right hand side (i.e., no loss in the measured density matrix elements in the original frame of reference) which in turn means no decoherence results.

* Contributed equally to this work.

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